MR3331402 (Review) 11F55 11F30 32M10 53C30

Yang, Jae-Hyun (KR-INHA)

Geometry and arithmetic on the Siegel-Jacobi space. (English. English summary)

Geometry and analysis on manifolds, 275–325, *Progr. Math.*, 308, *Birkhäuser/Springer, Cham*, 2015.

This paper gives a survey of definitions, facts and conjectures concerning the Siegel-Jacobi space $\mathbb{H}_{n,m} = \mathbb{H}_n \times \mathbb{C}^{m,n}$, where $\mathbb{H}_n$ is the Siegel half space, which is a non-symmetric homogeneous space for the Jacobi group $G^J = \mathrm{Sp}(n,\mathbb{R}) \ltimes H_{\mathbb{R}}^{(m,n)}$, $\mathrm{Sp}(n,\mathbb{R})$ is the usual symplectic group and

$$H_{\mathbb{R}}^{(m,n)} = \{(\lambda,\mu,\kappa) : \lambda,\mu \in \mathbb{R}^{(m,n)}, \kappa \in \mathbb{R}^{(m,m)}, \kappa + \mu^t\lambda \text{ symmetric}\}$$

is the adequate Heisenberg group.

There are 13 sections. In section 2, the author recalls Riemann metrics invariant under the classic G^J-action

$$(M,(\lambda,\mu,\kappa))\cdot(\Omega,Z) = ((A\Omega+B)(C\Omega+D)^{-1}, (Z+\lambda\Omega+\mu)(C\Omega+D)^{-1})$$

and their Laplacians. In Section 3, he discusses the Lie algebra of the Jacobi group and G^J-invariant differential operators on $\mathbb{H}_{n,m}$. Section 4 gives a description of the partial Cayley transform of the Siegel-Jacobi disk $\mathbb{D}_{m,n}$ onto the Siegel-Jacobi space, which gives a partially bounded realization of the Siegel-Jacobi space with a compatibility result of a partial Cayley transform. In Section 5, the author provides Riemannian metrics on the Siegel-Jacobi disk and their Laplacians, which are invariant under the natural transitive action of the suitable transform G_*^J of the Jacobi group. Section 6 shows a fundamental domain for the Siegel-Jacobi space with respect to the Siegel-Jacobi modular group. Section 7 contains the canonical automorphic factor for G^J, which is obtained by a geometrical method, and a review of the concept of Jacobi forms. Section 8 mentions singular Jacobi forms and their characterization in terms of a certain differential operator. In Section 9 the classical Siegel operator is extended to a Siegel-Jacobi operator and certain Hecke operators are discussed. Section 10 is devoted to the construction of vector-valued modular forms from Jacobi forms. This is done by the application of certain homogeneous pluriharmonic differential operators. A short discussion of Maass-Jacobi forms appears in Section 11 and basic facts concerning the Schrödinger-Weil representation are reported in Section 12. Finally, in Section 13, the author proposes several open problems about the geometry and arithmetic of the Siegel-Jacobi space.

Rolf Berndt