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Yang, Jae-Hyun (KR-INHA)
Holomorphic vector bundles over complex tori.
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Let E be a holomorphic vector bundle of rank r over a complex torus $T=\mathbf{C}^g/L$. The bundle E is said to be semihomogeneous if for each $x\in T$ there exists a line bundle F over T such that $T_x^*(E)\cong E\otimes F$, where T_x is the translation of T by x. The author proves that the following conditions are equivalent: (1) E is semihomogeneous; (2) the associated projective bundle $\mathbf{P}(E)$ is flat (i.e., admits a system of constant transition functions); (3) the total Chern class $c(E)$ is given by $c(E)=(1+c_1(E)/r)^r$; (4) the automorphy factor J of E is of the form $J(\alpha,z)=G(\alpha)\exp((\pi/r)H(z,\alpha)+(\pi/2r)H(\alpha,\alpha))$, where $\alpha\in L, z\in\mathbf{C}^n$, H is a Riemann form for T, and $G\colon L\to\mathrm{GL}(r,\mathbf{C})$ is a semirepresentation of L, i.e., $G(\alpha,\beta)=G(\alpha)G(\beta)\exp((i\pi/r)E(\beta,\alpha))$, where $E=\mathrm{Im}\,H$. The assertion (2) $\Rightarrow$ (4) was proved earlier by J. Hano [Nagoya Math. J. **61** (1976), 197–202; MR0419854]. Generalizing the result of Mukai and Oda for abelian varieties the author also proves that if E is simple (i.e. $H^0(T,\mathrm{End}\,E)=\mathbf{C}$), then (1), (2), (3) and (4) are equivalent to any of the following: (5) $\dim H^j(T,\mathrm{End}\,E)=\binom{g}{j}$ for all j, $j=1,2,\cdots,r$; (6) there exists an isogeny $f\colon\tilde T\to T$ and a line bundle L on $\tilde T$ such that $E=f_*(L)$.

D. N. Akhiezer